

السؤال الأول: ضع دائرة حول رمز الاجابة الصحيحة فيما يلي:

1- يُبسط المقدار المثلثي $[3 \cos^2 \theta + 3 \sin^2 \theta]$

- a) 3 b) 1 c) $3 \sec^2 \theta$ d) $1 - \tan^2 \theta$

2- المقدار المكافئ للمقدار المثلثي $[\cos^2 \theta + \tan^2 \theta \cos^2 \theta]$ هو

- a) $\cos \theta$ b) 1 c) $\sec^2 \theta$ d) $1 - \tan^2 \theta$

3- احد الاتية يكافئ المقدار المثلثي $[\csc \theta]$ هو

- a) $\sin \theta \cot \theta$ b) $\cos \theta \cot \theta$ c) $\sec \theta \cot \theta$ d) $\cos \theta \tan \theta$

4- تبسط المقادير المثلثية الاتية بحيث تساوي (1) عدا

- a) $\sec^2 x(1 + \cot^2 x)$ b) $\frac{\cos \theta}{\sec \theta} + \frac{\sin \theta}{\csc \theta}$ c) $\cos^2 x(1 + \tan^2 x)$ d) $\sin^2 x(1 + \cot^2 x)$

5- المقدار المكافئ للمقدار المثلثي $[(\cos \frac{\pi}{2} - x) \tan x - \sec x]$ هو

- a) $\sec x$ b) $\cos x$ c) $-\sec x$ d) $-\cos x$

6- يُبسط المقدار المثلثي $[\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)}]$

- a) $\sin \theta - \cos \theta$ b) $\cos \theta - \sin \theta$ c) $\sin \theta + \cos \theta$ d) $-\sin \theta - \cos \theta$

7- المقدار المكافئ للمقدار المثلثي $[\frac{(1 - \cos \theta)(1 + \cos \theta)}{\cos^2 \theta}]$ هو

- a) $\tan^2 \theta$ b) 1 c) $\cot^2 \theta$ d) 0

8- يُبسط المقدار المثلثي $[\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x}]$

- a) $\csc x$ b) $\sec x$ c) $2 \sec x$ d) $2 \csc x$

9- احد الاتية يكافئ المقدار المثلثي $[(\cos x + \sin x)^2]$ هو

- a) $1 + \sin 2x$ b) $1 - \sin 2x$ c) $1 + 2 \sin x$ d) $1 - 2 \sin x$

10- يمكن كتابة المقدار المثلثي $[\frac{1}{1 - \cos x}]$ بحيث لا يحوي كسراً

- a) $\sec^2 x - \tan x \sec x$ b) $\sec^2 x + \tan x \sec x$ c) $\csc^2 x - \cot x \csc x$ d) $\csc^2 x + \cot x \csc x$

☒ اذا كان $[\cos(x - \frac{\pi}{2}) = \frac{3}{5}]$ ، $\frac{\pi}{2} < x < \pi$ اجب عن الفقرات (11،12،13)

11- قيمة $[\sin(x - \frac{\pi}{2})]$ هي

- a) $\frac{-3}{5}$ b) $\frac{-4}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

12- قيمة $[\sec x]$ هي

- a) $\frac{5}{3}$ b) $\frac{5}{4}$ c) $\frac{-5}{3}$ d) $\frac{-5}{4}$

13- قيمة $[\cot -x]$ هي

- a) $\frac{-3}{4}$ b) $\frac{-4}{3}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$

14- قيمة المقدار $[\ln|\sec x + \tan x| + \ln|\sec x - \tan x|]$ هي

- a) 0 b) 1 c) -1 d) غير معرف

15- قيمة $[\cot 165]$ هي

- a) $2 - \sqrt{3}$ b) $2 + \sqrt{3}$ c) $\sqrt{3} - 2$ d) $-\sqrt{3} - 2$

☒ اذا كان $[\sin \alpha = \frac{4}{5}]$ ، $0 < \alpha < \frac{\pi}{2}$ و $[\cos \beta = \frac{-12}{13}]$ ، $\pi < \beta < \frac{3\pi}{2}$ اجب عن الفقرتين (16،17)

16- قيمة $[\cos(\alpha - \beta)]$ هي

- a) $\frac{-63}{65}$ b) $\frac{16}{65}$ c) $\frac{56}{65}$ d) $\frac{-56}{65}$

17- قيمة $[\cot(\alpha - \beta)]$ هي

- a) $\frac{63}{16}$ b) $\frac{16}{63}$ c) $\frac{56}{33}$ d) $\frac{33}{65}$

☒ إذا كان $[\sin x = \frac{-1}{\sqrt{2}}]$ ، $\pi < x < \frac{3\pi}{2}$] اجب عن الفقرتين (18،19)

18- قيمة $[\cos 2x]$ هي

- a) 1 b) 0 c) -1 d) $\frac{-1}{\sqrt{2}}$

19- قيمة $[\csc 2x]$ هي

- a) 1 b) 0 c) -1 d) $\sqrt{2}$

☒ إذا كان $[\cos \theta = \frac{-3}{5}]$ ، $\pi < \theta < \frac{3\pi}{2}$] اجب عن الفقرات (20،21،22،23،24،25)

20- قيمة $[\sin 2\theta]$ هي

- a) $\frac{24}{25}$ b) $\frac{-24}{25}$ c) $\frac{-7}{25}$ d) $\frac{7}{25}$

21- قيمة $[\tan 2\theta]$ هي

- a) $\frac{-27}{40}$ b) $\frac{-24}{5}$ c) $\frac{-40}{27}$ d) $\frac{-5}{24}$

22- قيمة $[\sin \frac{\theta}{2}]$ هي

- a) $\frac{-\sqrt{5}}{5}$ b) $\frac{\sqrt{5}}{5}$ c) $\frac{-2\sqrt{5}}{5}$ d) $\frac{2\sqrt{5}}{5}$

23- قيمة $[\cos \frac{\theta}{2}]$ هي

- a) $\frac{-\sqrt{5}}{5}$ b) $\frac{\sqrt{5}}{5}$ c) $\frac{-2\sqrt{5}}{5}$ d) $\frac{2\sqrt{5}}{5}$

24- قيمة $[\tan \frac{\theta}{2}]$ هي

- a) $\frac{-1}{2}$ b) $\frac{1}{2}$ c) -2 d) 2

25- قيمة $[\sin 2\theta \tan \frac{\theta}{2}]$ هي

- a) 1 b) -1 c) $\frac{48}{25}$ d) $\frac{-48}{25}$

26- يمكن كتابة المقدار المثلثي $[\cos 3\theta]$ على صورة $[\cos \theta]$ على النحو التالي

- a) $4 \cos^3 \theta + 3 \cos \theta$ b) $2 \cos^3 \theta - 3 \cos \theta$ c) $4 \cos^3 \theta - 3 \cos \theta$ d) $4 \cos^3 \theta - \cos \theta$

27- يمكن كتابة المقدار المثلثي $[\sin^4 x \cos^2 x]$ بدلالة القوة الأولى لجيب التمام على النحو التالي

- a) $\frac{1}{16} (1 - \cos 2x - \cos 4x + 2 \cos x \cos 2x)$ b) $\frac{1}{16} (\cos 2x - \cos 4x + \cos 2x \cos 4x)$
c) $\frac{1}{16} (1 - \cos 2x - \cos 4x + \cos 2x \cos 4x)$ d) $\frac{1}{16} (1 - \cos 2x - \cos 2x \cos 4x)$

28- قيمة $[\tan 22.5^\circ]$ هي

- a) $\sqrt{\frac{1-\sqrt{2}}{1+\sqrt{2}}}$ b) $-\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ c) $-\sqrt{\frac{1-\sqrt{2}}{1+\sqrt{2}}}$ d) $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$

29- يمكن كتابة المقدار المثلثي $[\cos 3x \sin 5x]$ في صورة مجموع او فرق على النحو التالي

- a) $-\frac{1}{2} [\cos 2x + \cos 8x]$ b) $-\frac{1}{2} [-\sin 2x - \sin 8x]$
c) $\frac{1}{2} [\cos 2x - \cos 8x]$ d) $\frac{1}{2} [\sin 8x - \sin 2x]$

30- يمكن كتابة المقدار المثلثي $[\sin 2x - \sin 6x]$ في صورة ضرب على النحو التالي

- a) $-2 \cos(2x) \sin(4x)$ b) $2 \sin(4x) \cos(2x)$
c) $-2 \cos(4x) \sin(2x)$ d) $2 \sin(2x) \cos(4x)$

31- المقدار المكافئ للمقدار المثلثي $[\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x}]$ هو

- a) $\tan^2 x$ b) $2 \tan 2x$ c) $\tan 2x$ d) $2 \tan x$

32- حل المعادلة المثلثية $[\cos x = -\frac{1}{\sqrt{2}}]$ في الفترة $[0, 2\pi]$ هو

- a) $\frac{3\pi}{4}, \frac{7\pi}{4}$ b) $\frac{3\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi$ c) $\frac{3\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi$ d) $\frac{3\pi}{4}, \frac{5\pi}{4}$

33- اذا كان $[\tan^{-1} - 2 = -1.1]$ فإن حل المعادلة المثلثية $[\tan x = -2]$ هو

- a) $1.1 + k\pi$ b) $1.1 + 2k\pi$ c) $-1.1 + k\pi$ d) $-1.1 + 2k\pi$

34- اذا كان $[\sin^{-1} \frac{1}{3} = 0.34, \pi = 3.14]$ فإن حل المعادلة المثلثية $[4 \sin \beta + 1 = \sin \beta]$ في الفترة $[0, 2\pi]$ هو

- a) $\beta = 2.80$ او 5.94 b) $\beta = 3.34$ او 5.94 c) $\beta = 2.80$ او 3.34 d) $\beta = 2.80$ او 6.62

35- حل المعادلة المثلثية $[2 \cos x \sin x = -\sin x]$ في الفترة $[0, 2\pi]$ هو

- a) $(0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}, 2\pi)$ b) $(0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi)$ c) $(0, \pi, \frac{\pi}{3}, \frac{4\pi}{3}, 2\pi)$ d) $(\pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi)$

36- احد الاتية يعد حل للمعادلة المثلثية $[\cos x \sin x = \frac{1}{2}]$ في الفترة $[0, 2\pi]$ هو

- a) $(\frac{\pi}{4})$ b) $(\frac{5\pi}{4})$ c) $(\frac{\pi}{2})$ d) $(\frac{3\pi}{4})$

37- عدد حلول المعادلة المثلثية $[\cos x - 1 = \sin x]$ في الفترة $[0, 2\pi]$ هو

- a) 1 b) 2 c) 3 d) 4

38- حل المعادلة المثلثية $[\frac{1+\cos x}{1-\cos x} = 3]$ في الفترة $[0, 2\pi]$ هو

- a) $(\frac{\pi}{3}, \frac{2\pi}{3})$ b) $(\frac{2\pi}{3}, \frac{4\pi}{3})$ c) $(\frac{\pi}{3}, \frac{5\pi}{3})$ d) $(\frac{2\pi}{3}, \frac{5\pi}{3})$

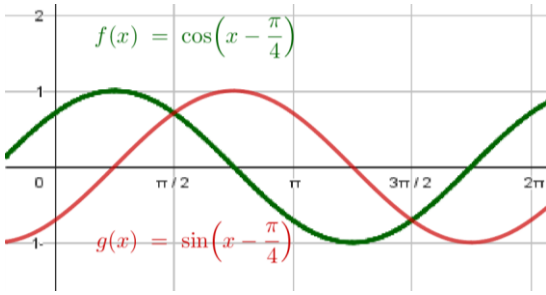
39- حل المعادلة المثلثية $[2 \cos \frac{x}{2} - 1 = 0]$ في الفترة $[0, 2\pi]$ هو

- a) $(\frac{\pi}{3}, \frac{5\pi}{3})$ b) $(\frac{5\pi}{3}, \frac{10\pi}{3})$ c) $(\frac{\pi}{3}, \frac{10\pi}{3})$ d) $(\frac{2\pi}{3}, \frac{10\pi}{3})$

40- بالاستعانة بالشكل المجاور حل المعادلة المثلثية $[\cos(x - \frac{\pi}{4}) - \sin(x - \frac{\pi}{4}) = 0]$

في الفترة $[0, 2\pi]$ هو

- a) $(0, \pi, 2\pi)$ b) $(\frac{\pi}{2}, \frac{3\pi}{2})$
c) $(0, \pi, \frac{\pi}{2}, \frac{3\pi}{2})$ d) $(\pi, \frac{\pi}{2}, \frac{3\pi}{2})$



السؤال الثاني: اثبت صحة المتطابقات الآتية:

- 1- $\ln|\cos x| = \frac{1}{2}(\ln|1 + \cos 2x| - \ln|2|)$
- 2- $4(\sin^6 x + \cos^6 x) = 4 - 3 \sin^2 2x$
- 3- $\tan^2 x \cos x = \tan x \sin x$
- 4- $\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$

السؤال الثالث: حل المعادلات المثلثية الآتية في الفترة $[0, 2\pi]$:

- 1- $\sin x + 2 \sin x \cos x = 0$
- 2- $\tan \frac{x}{2} - \sin x = 0$
- 3- $\sin(\cos x) = 0$
- 4- $4 \sin^2 x - 8 \sin x + 3 = 0$
- 5- $\cos^2 x + 3 \cos x + 2 = 0$
- 6- $2 \cos^2 x + 3 \sin x = 0$

مع تحيات لؤي مرعي

$$\begin{aligned}
 1- & 3 \cos^2 \theta + 3 \sin^2 \theta \\
 &= 3 (\cos^2 \theta + \sin^2 \theta) \quad (\cos^2 \theta + \sin^2 \theta = 1) \\
 &= 3(1) = 3 \quad (a)
 \end{aligned}$$

$$\begin{aligned}
 2- & \cos^2 \theta + \tan^2 \theta \cos^2 \theta \\
 &= \cos^2 \theta (1 + \tan^2 \theta) \\
 &= \cos^2 \theta \sec^2 \theta \\
 &= \cos^2 \theta \frac{1}{\cos^2 \theta}, \sec \theta = \frac{1}{\cos \theta} \\
 &= 1 \quad (b)
 \end{aligned}$$

$$3- \sec \theta \cot \theta = \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta} = \csc \theta$$

نجرب الخيارات واحدا تلو الآخر للوصول الى الخيار الصحيح وهو (c)

$$\begin{aligned}
 4- & \sec^2 x (1 + \cot^2 x) = \sec^2 x \csc^2 x \\
 &= \frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x \sin^2 x} \neq 1
 \end{aligned}$$

نجرب الخيارات واحدا تلو الآخر للوصول الى الخيار الصحيح وهو (a)

$$\begin{aligned}
 5- & (\cos \frac{\pi}{2} - x) \tan x - \sec x = \sin x \tan x - \sec x = \sin x \frac{\sin x}{\cos x} - \sec x \\
 &= \frac{\sin^2 x}{\cos x} - \frac{1}{\cos x} = \\
 &= \frac{\sin^2 x - 1}{\cos x} \\
 &= -\frac{\cos^2 x}{\cos x} = -\cos x \quad (d)
 \end{aligned}$$

$$\begin{aligned}
 6- & \frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} \\
 &= \frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \frac{(-\sin(\theta))^2 - \cos^2(\theta)}{-\sin(\theta) - \cos(\theta)} = \frac{\sin^2(\theta) - \cos^2(\theta)}{-\sin(\theta) - \cos(\theta)} \\
 &= \frac{(\sin x - \cos x)(\sin x + \cos x)}{-(\sin x + \cos x)} \\
 &= -(\sin x + \cos x) = \cos x - \sin x \quad (b)
 \end{aligned}$$

$$7- \frac{(1 - \cos \theta)(1 + \cos \theta)}{\cos^2 \theta} = \frac{(1 - \cos^2 \theta)}{\cos^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta \quad (a)$$

$$8- \frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} \quad \text{توحيد المقامات}$$

$$\begin{aligned}
&= \frac{\sin^2 x + (1 + \cos x)^2}{\sin x(1 + \cos x)} \\
&= \frac{\sin^2 x + 1 + 2 \cos x + \cos^2 x}{\sin x(1 + \cos x)} \\
&= \frac{(\sin^2 x + \cos^2 x) + 1 + 2 \cos x}{\sin x(1 + \cos x)} = \frac{2 + 2 \cos x}{\sin x(1 + \cos x)} = \frac{2(1 + \cos x)}{\sin x(1 + \cos x)} = \frac{2}{\sin x} = 2 \csc x \quad (d)
\end{aligned}$$

$$\begin{aligned}
9- (\cos x + \sin x)^2 &= (\cos x - \sin x)^2 = (\cos^2 x + \sin^2 x) - 2 \sin x \cos x \\
&= 1 - 2 \sin x \cos x = 1 - \sin 2x \quad (b)
\end{aligned}$$

$$\begin{aligned}
10- \frac{1}{1 - \cos x} &\text{ بالضرب بالمرافق} \\
&= \frac{1}{1 - \cos x} \times \frac{(1 + \cos x)}{(1 + \cos x)} = \frac{(1 + \cos x)}{(1 - \cos^2 x)}, \quad 1 - \cos^2 x = \sin^2 x \\
\frac{(1 + \cos x)}{(\sin^2 x)} &= \frac{1}{(\sin^2 x)} + \frac{\cos x}{(\sin^2 x)} = \frac{1}{(\sin^2 x)} + \left(\frac{\cos x}{(\sin x)} \times \frac{1}{\sin x} \right) \\
&= \csc^2 x + \cot x \csc x \quad (d)
\end{aligned}$$

$$\begin{aligned}
\boxtimes \cos\left(x - \frac{\pi}{2}\right) &= \frac{3}{5} \frac{\pi}{2} < x < \pi, \therefore \cos\left(x - \frac{\pi}{2}\right) = \cos\left(-\left(\frac{\pi}{2} - x\right)\right) = \cos\left(\frac{\pi}{2} - x\right) = \sin x = \frac{3}{5} \\
\therefore \sin^2 x + \cos^2 x &= 1 \Rightarrow \cos^2 x = 1 - \sin^2 x \Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2 \Rightarrow \cos^2 x = \frac{25-9}{25} \\
\therefore \cos^2 x &= \frac{16}{25} \Rightarrow \cos x = \pm \frac{4}{5} \Rightarrow \cos x = -\frac{4}{5}, \frac{\pi}{2} < x < \pi \quad (\text{الزاوية بالربع الثاني})
\end{aligned}$$

$$11- \sin\left(x - \frac{\pi}{2}\right) = \sin\left(-\left(\frac{\pi}{2} - x\right)\right) = -\sin\left(\frac{\pi}{2} - x\right) = -\cos x = -\left(\frac{-4}{5}\right) = \frac{4}{5} \quad (d)$$

$$12- \sec x = \frac{1}{\cos x} = \frac{1}{\frac{-4}{5}} = 1 \times \frac{-5}{4} = \frac{-5}{4} \quad (d)$$

$$13- \cot -x = -\cot x = -\frac{\cos x}{\sin x} = -\left(\frac{-4}{\frac{3}{5}}\right) = -\left(\frac{-4}{5} \times \frac{5}{3}\right) = \frac{4}{3} \quad (d)$$

$$\begin{aligned}
14- \ln|\sec x + \tan x| + \ln|\sec x - \tan x| &= \ln|(\sec x + \tan x)(\sec x - \tan x)| = \ln|\sec^2 x - \tan^2 x| \\
&= \ln|1| = 0, \quad \sec^2 x = 1 + \tan^2 x \quad (a)
\end{aligned}$$

$$15- \cot 165 = -\cot 15 = -\left(\frac{\cos 15}{\sin 15}\right), \quad 15^\circ = 60^\circ - 45^\circ$$

$$\begin{aligned}
\therefore \cos(a - b) &= (\cos a \cos b + \sin a \sin b) = \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ = \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) \\
&= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}
\end{aligned}$$

$$\begin{aligned}
\therefore \sin(a - b) &= (\sin a \cos b - \cos a \sin b) = \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ = \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right) \\
&= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}
\end{aligned}$$

$$\begin{aligned}
\therefore \cot 165 &= -\cot 15 = -\left(\frac{\cos 15}{\sin 15}\right) = -\left(\frac{\frac{1+\sqrt{3}}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}\right) = -\left(\frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{3}-1}\right) = -\left(\frac{1+\sqrt{3}}{\sqrt{3}-1}\right) = -\left(\frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\right) = -\left(\frac{1+3+2\sqrt{3}}{3-1}\right) \\
&= -\left(\frac{4+2\sqrt{3}}{2}\right) = -\left(\frac{4}{2} + \frac{2\sqrt{3}}{2}\right) = -(2 + \sqrt{3}) = -2 - \sqrt{3} \quad (d)
\end{aligned}$$

$$\boxed{\times} \sin \alpha = \frac{4}{5}, 0 < \alpha < \frac{\pi}{2}$$

$$\cos \beta = \frac{-12}{13}, \pi < \beta < \frac{3\pi}{2}$$

$$\Rightarrow \therefore \cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{4}{5}\right)^2 = \frac{25-16}{25} = \frac{9}{25}$$

$$\therefore \cos \alpha = \pm \frac{3}{5}, \therefore \cos \alpha = \frac{3}{5}, 0 < \alpha < \frac{\pi}{2} \text{ (الزاوية بالرّبع الاول)}$$

$$\Rightarrow \therefore \sin^2 \beta = 1 - \cos^2 \beta = 1 - \left(\frac{-12}{13}\right)^2 = \frac{169-144}{169} = \frac{25}{169}$$

$$\therefore \sin \beta = \pm \frac{5}{13}, \therefore \sin \beta = \frac{-5}{13}, \pi < \beta < \frac{3\pi}{2} \text{ (الزاوية بالرّبع الثالث)}$$

$$\Rightarrow \therefore \tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \left(\frac{\frac{4}{5}}{\frac{3}{5}}\right) = \frac{4}{5} \times \frac{5}{3} = \frac{4}{3}$$

$$\Rightarrow \therefore \tan \beta = \frac{\sin \beta}{\cos \beta} = \left(\frac{\frac{-5}{13}}{\frac{-12}{13}}\right) = \frac{-5}{13} \times \frac{13}{-12} = \frac{5}{12}$$

$$16- \cos(\alpha - \beta) = (\cos \alpha \cos \beta + \sin \alpha \sin \beta) = \left(\frac{3}{5} \times \frac{-12}{13}\right) + \left(\frac{4}{5} \times \frac{-5}{13}\right) = \left(\frac{-36}{65}\right) + \left(\frac{-20}{65}\right) = \frac{-36-20}{65}$$

$$\cos(\alpha - \beta) = \frac{-56}{65} \text{ (d)}$$

$$17- \cot(\alpha - \beta) = \frac{1}{\tan(\alpha - \beta)}$$

$$\therefore \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{\frac{4}{3} - \frac{5}{12}}{1 + \left(\frac{4}{3} \times \frac{5}{12}\right)} = \frac{\frac{16-5}{12}}{1 + \frac{20}{36}} = \frac{\frac{11}{12}}{\frac{36+20}{36}} = \frac{\frac{11}{12}}{\frac{56}{36}} = \frac{11}{12} \times \frac{36}{56} = \frac{33}{56}$$

$$\therefore \cot(\alpha - \beta) = \frac{1}{\tan(\alpha - \beta)} = \frac{1}{\frac{33}{56}} = \frac{56}{33} \text{ (c)}$$

$$\boxed{\times} \sin x = \frac{-1}{\sqrt{2}}, \pi < x < \frac{3\pi}{2} \text{ (الزاوية بالرّبع الثالث)}$$

$$\therefore \sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x \Rightarrow \cos^2 x = 1 - \left(\frac{-1}{\sqrt{2}}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \cos x = \pm \frac{1}{\sqrt{2}}, \therefore \cos x = -\frac{1}{\sqrt{2}}, \pi < x < \frac{3\pi}{2} \text{ (الزاوية بالرّبع الثالث)}$$

$$18- \cos 2x = 1 - 2 \sin^2 x = 1 - \left(2 \times \frac{1}{2}\right) = 1 - 1 = 0 \text{ (b)}$$

$$19- \csc 2x = \frac{1}{\sin 2x} \Rightarrow \therefore \sin 2x = 2 \sin x \cos x = 2 \times \frac{-1}{\sqrt{2}} \times \frac{-1}{\sqrt{2}} = 1$$

$$\therefore \csc 2x = \frac{1}{\sin 2x} = \frac{1}{1} = 1 \text{ (a)}$$

$$\boxed{\times} \cos \theta = \frac{-3}{5}, \pi < \theta < \frac{3\pi}{2} \text{ (الزاوية بالرّبع الثالث)}$$

$$\Rightarrow \therefore \sin^2 \theta = 1 - \cos^2 \theta = 1 - \left(\frac{-3}{5}\right)^2 = \frac{25-9}{25} = \frac{16}{25}$$

$$\therefore \sin \theta = \pm \frac{4}{5} \Rightarrow \sin \theta = \frac{-4}{5}, \pi < \theta < \frac{3\pi}{2} \text{ (الزاوية بالرّبع الثالث)}$$

$$20- \sin 2\theta = 2 \sin \theta \cos \theta = 2 \times \frac{-4}{5} \times \frac{-3}{5} = \frac{24}{25} \text{ (a)}$$

$$21- \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\Rightarrow \therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{-4}{5} \times \frac{5}{-3} = \frac{4}{3}$$

$$\Rightarrow \therefore \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \times \frac{4}{3}}{1 - \left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{1 - \frac{16}{9}} = \frac{\frac{8}{3}}{\frac{9-16}{9}} = \frac{\frac{8}{3}}{\frac{-5}{9}} = \frac{8}{3} \times \frac{9}{-5} = \frac{-72}{15} = \frac{-24}{5} \text{ (b)}$$

$$22- \sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}, \pi < \theta < \frac{3\pi}{2} \text{ (الزاوية بالرّبع الثالث)}, \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4} \text{ , بالربع الثاني , (} \frac{\theta}{2} \text{) الزاوية}$$

$$\therefore \sin \frac{\theta}{2} = +\sqrt{\frac{1-\cos \theta}{2}} = \sqrt{\frac{1-\frac{-3}{5}}{2}} = \sqrt{\frac{5-(-3)}{10}} = \sqrt{\frac{5+3}{10}} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \frac{\sqrt{4}}{\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \quad (d)$$

23- $\cos \frac{\theta}{2} = \pm \sqrt{\frac{1+\cos \theta}{2}}$, $\pi < \theta < \frac{3\pi}{2}$, (الزاوية بالرابع الثالث) , $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$, بالربع الثاني (الزاوية $\frac{\theta}{2}$)

$$\therefore \cos \frac{\theta}{2} = -\sqrt{\frac{1+\cos \theta}{2}} = -\sqrt{\frac{1+\frac{-3}{5}}{2}} = -\sqrt{\frac{5+(-3)}{10}} = -\sqrt{\frac{5-3}{10}} = -\sqrt{\frac{2}{10}} = -\sqrt{\frac{1}{5}} = -\frac{\sqrt{1}}{\sqrt{5}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5} \quad (a)$$

24- $\tan \frac{\theta}{2} = \pm \sqrt{\frac{1-\cos \theta}{1+\cos \theta}}$, $\pi < \theta < \frac{3\pi}{2}$, (الزاوية بالرابع الثالث) , $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$, بالربع الثاني (الزاوية $\frac{\theta}{2}$)

$$\therefore \tan \frac{\theta}{2} = -\sqrt{\frac{1-\cos \theta}{1+\cos \theta}} = -\sqrt{\frac{1-\frac{-3}{5}}{1+\frac{-3}{5}}} = -\sqrt{\frac{5-(-3)}{5+(-3)}} = -\sqrt{\frac{5+3}{5-3}} = -\sqrt{\frac{8}{2}} = -\sqrt{4} = -2 \quad (c)$$

25- $\sin 2\theta \tan \frac{\theta}{2} = \frac{24}{25} \times -2 = \frac{-48}{25} \quad (d)$

26- $\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$

$$= (2 \cos^2 \theta - 1) \cos \theta - (2 \sin \theta \cos \theta) \sin \theta , \cos 2\theta = 2 \cos^2 \theta - 1 , \sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^3 \theta - \cos \theta - 2 \cos \theta \sin^2 \theta = 2 \cos^3 \theta - \cos \theta - 2 \cos \theta (1 - \cos^2 \theta) , \sin^2 \theta = 1 - \cos^2 \theta$$

$$= 2 \cos^3 \theta - \cos \theta - 2 \cos \theta + \cos^3 \theta = 4 \cos^3 \theta - 3 \cos \theta \quad (c)$$

27- $\sin^4 x \cos^2 x = \sin^2 x (\sin^2 x \cos^2 x)$

$$= \sin^2 x \sin^2 x \cos^2 x = \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1+\cos 2x}{2}\right) , \sin^2 x = \frac{1-\cos 2x}{2} , \cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1-\cos^2 2x}{4}\right) = \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1}{4} - \frac{\cos^2 2x}{4}\right) = \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1}{4} - \frac{1}{4} \cos^2 2x\right)$$

$$= \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1}{4} - \frac{1}{4} \left(\frac{1+\cos 4x}{2}\right)\right) , \cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1}{4} - \frac{1}{8} - \left(\frac{\cos 4x}{8}\right)\right) = \left(\frac{1-\cos 2x}{2}\right) \left(\frac{2-1}{8} - \left(\frac{\cos 4x}{8}\right)\right) = \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1}{8} - \left(\frac{\cos 4x}{8}\right)\right)$$

$$= \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1}{8} (1 - \cos 4x)\right) = \frac{1}{16} ((1 - \cos 2x)(1 - \cos 4x))$$

$$= \frac{1}{16} (1 - \cos 2x - \cos 4x + \cos 2x \cos 4x) \quad (c)$$

28- $\tan 22.5^\circ = \tan \frac{45^\circ}{2} = \sqrt{\frac{1-\cos \theta}{1+\cos \theta}}$, $0 < \frac{\theta}{2} < \frac{\pi}{2}$, (الزاوية $\frac{\theta}{2}$) بالربع الاول

$$= \tan \frac{45^\circ}{2} = \sqrt{\frac{1-\cos 45^\circ}{1+\cos 45^\circ}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}} = \sqrt{\frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} \quad (d)$$

29- $\cos 3x \sin 5x = -\frac{1}{2} [\sin(\alpha - \beta) - \sin(\alpha + \beta)] = -\frac{1}{2} [\sin(3x - 5x) - \sin(3x + 5x)]$
 $= -\frac{1}{2} [\sin(-2x) - \sin(8x)] \quad (b)$

30- $\sin 2x - \sin 6x = 2 \cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right) = 2 \cos\left(\frac{2x+6x}{2}\right) \sin\left(\frac{2x-6x}{2}\right) = 2 \cos\left(\frac{8x}{2}\right) \sin\left(\frac{-4x}{2}\right)$
 $= 2 \cos 4x \sin -2x = -2 \cos 4x \sin 2x \quad (c)$

31- $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{\cos^2 x - \sin^2 x} = \frac{(\cos^2 x + \sin^2 x + 2 \cos x \sin x) - (\cos^2 x + \sin^2 x - 2 \cos x \sin x)}{\cos^2 x - \sin^2 x}$
 $= \frac{\cos^2 x + \sin^2 x + 2 \cos x \sin x - \cos^2 x - \sin^2 x + 2 \cos x \sin x}{\cos^2 x - \sin^2 x} = \frac{4 \cos x \sin x}{\cos^2 x - \sin^2 x} , \cos 2x = \cos^2 x - \sin^2 x$
 $= \frac{2(2 \cos x \sin x)}{\cos 2x} , \sin 2x = 2 \sin x \cos x$
 $= \frac{2 \sin 2x}{\cos 2x} = 2 \tan 2x \quad (b)$

32- $\cos x = -\frac{1}{\sqrt{2}} , [0, 2\pi]$

$$\therefore \cos x = \left| -\frac{1}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}} \Rightarrow x = 45^\circ = \frac{\pi}{4} , \text{ سالب بالربع الثاني والثالث } (\cos x)$$

$$\Rightarrow x = \pi - \frac{\pi}{4} = \frac{4\pi - \pi}{4} = \frac{3\pi}{4} , \text{ الربع الثاني}$$

$$\Rightarrow x = \pi + \frac{\pi}{4} = \frac{4\pi + \pi}{4} = \frac{5\pi}{4} , \text{ الربع الثالث } \Rightarrow x = \frac{3\pi}{4} , x = \frac{5\pi}{4} \quad (d)$$

33- $\tan x = -2 \Rightarrow x = \tan^{-1} -2 = -1.1$, من السؤال

$\therefore \tan x \in [-\frac{\pi}{2}, \frac{\pi}{2}] , [-\frac{\pi}{2}, 0]$ سالب بالفترة $(\tan x)$

$\Rightarrow \therefore x = -1, 1 + k\pi$ (c)

34- $4 \sin \beta + 1 = \sin \beta$, $\sin^{-1} \frac{1}{3} = 0.34$, $\pi = 3.14$, $[0, 2\pi]$

$4 \sin \beta + 1 = \sin \beta \Rightarrow 4 \sin \beta - \sin \beta = -1 \Rightarrow 3 \sin \beta = -1 \Rightarrow \sin \beta = \frac{-1}{3}$

$\therefore \sin \beta = \left| \frac{-1}{3} \right| = \frac{1}{3} \Rightarrow \beta = \sin^{-1} \frac{1}{3} = 0.34$, سالب بالربع الثالث والرابع $(\sin \beta)$

$\Rightarrow \beta = \pi + 0.34 = 3.48$, $\in [0, 2\pi]$ الربع الثالث

$\Rightarrow \beta = 2\pi - 0.34 = 5.94$, $\in [0, 2\pi]$ الربع الرابع $\Rightarrow \beta = 3.48, \beta = 5.94$ (b)

35- $2 \cos x \sin x = -\sin x$, $[0, 2\pi]$

$2 \cos x \sin x + \sin x = 0 \Rightarrow \sin x (2 \cos x + 1) = 0$

$\therefore \sin x = 0 \Rightarrow x = 0 \text{ or } \pi \text{ or } 2\pi \in [0, 2\pi]$

or $\therefore 2 \cos x + 1 = 0 \Rightarrow \cos x = \frac{-1}{2} \Rightarrow \therefore \cos x = \left| -\frac{1}{2} \right| = \frac{1}{2} \Rightarrow x = 60^\circ = \frac{\pi}{3}$, سالب بالربع الثاني والثالث $(\cos x)$

$\Rightarrow x = \pi - \frac{\pi}{3} = \frac{3\pi - \pi}{3} = \frac{2\pi}{3}$, الربع الثاني

$\Rightarrow x = \pi + \frac{\pi}{3} = \frac{3\pi + \pi}{3} = \frac{4\pi}{3}$, الربع الثالث $\Rightarrow \therefore x = \frac{2\pi}{3}, x = \frac{4\pi}{3} \in [0, 2\pi]$

$\therefore x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi \in [0, 2\pi]$ (b)

36- $\cos x \sin x = \frac{1}{2}$, $[0, 2\pi]$

$\therefore \cos x \sin x = \frac{1}{2} \Rightarrow 2 \cos x \sin x = 1 \Rightarrow \sin 2x = 1$, $\sin 2x = 2 \sin x \cos x$

$\therefore 2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$, $\in [0, 2\pi]$, موجب بالربع الأول والثاني $(\sin 2x)$ (a)

37- $\cos x - 1 = \sin x$, $[0, 2\pi]$

$(\cos x - 1)^2 = (\sin x)^2$ بتربيع الطرفين

$\Rightarrow \cos^2 x - 2 \cos x + 1 = \sin^2 x \Rightarrow \cos^2 x - \sin^2 x - 2 \cos x + 1 = 0$

$\Rightarrow \cos^2 x - (1 - \cos^2 x) - 2 \cos x + 1 = 0$, $\sin^2 x = 1 - \cos^2 x$

$\Rightarrow \cos^2 x - 1 + \cos^2 x - 2 \cos x + 1 = 0 \Rightarrow 2 \cos^2 x - 2 \cos x = 0 \Rightarrow 2 \cos x (\cos x - 1) = 0$

$\therefore 2 \cos x = 0 \Rightarrow \cos x = 0 \Rightarrow \therefore x = 90^\circ = \frac{\pi}{2}, 270^\circ = \frac{3\pi}{2}$ نعوض قيم (x) في المعادلة المثلثية الرئيسية

$\therefore \cos \frac{\pi}{2} - 1 = \sin \frac{\pi}{2} \Rightarrow -1 \neq 1 \therefore x = \frac{\pi}{2}$ مرفوضة لا تحقق المعادلة

$\therefore \cos \frac{3\pi}{2} - 1 = \sin \frac{3\pi}{2} \Rightarrow -1 = -1 \therefore x = \frac{3\pi}{2}$ تحقق المعادلة

or $\therefore \cos x - 1 = 0 \Rightarrow \cos x = 1 \Rightarrow \therefore \cos x = 1 \Rightarrow x = 0^\circ = 0, 360^\circ = 2\pi$ في المعادلة المثلثية الرئيسية نعوض قيم (x)

$\therefore \cos 0 - 1 = \sin 0 \Rightarrow 1 - 1 = 0 \therefore x = 0$ تحقق المعادلة

$\therefore \cos 2\pi - 1 = \sin 2\pi \Rightarrow 1 - 1 = 0 \therefore x = 2\pi$ تحقق المعادلة

(c) عدد الحلول هو (3)

38- $\frac{1 + \cos x}{1 - \cos x} = 3$, $[0, 2\pi]$

$1 + \cos x = 3(1 - \cos x) \Rightarrow 1 + \cos x = 3 - 3 \cos x \Rightarrow \cos x + 3 \cos x = 3 - 1 \Rightarrow 4 \cos x = 2$

$\Rightarrow \cos x = \frac{2}{4} = \frac{1}{2} \Rightarrow x = 60^\circ = \frac{\pi}{3}$, موجب بالربع الأول والرابع $(\cos x)$

$\Rightarrow x = 2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$, الربع الرابع $\Rightarrow \therefore x = \frac{\pi}{3}, x = \frac{5\pi}{3} \in [0, 2\pi]$ (c)

39- $2 \cos \frac{x}{2} - 1 = 0$, $[0, 2\pi]$

$2 \cos \frac{x}{2} - 1 = 0 \Rightarrow 2 \cos \frac{x}{2} = 1 \Rightarrow \cos \frac{x}{2} = \frac{1}{2}$

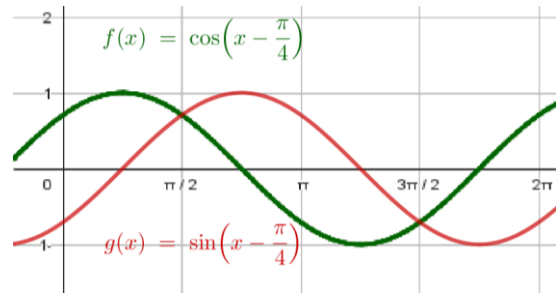
$\Rightarrow \therefore \frac{x}{2} = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{3} \times 2 = \frac{2\pi}{3}$

$$\Rightarrow \therefore x = \left(2\pi - \frac{\pi}{3}\right) \times 2 = \left(\frac{6\pi - \pi}{3}\right) \times 2 = \frac{10\pi}{3}, \therefore x = \frac{2\pi}{3}, \frac{10\pi}{3} \in [0, 2\pi], \text{ موجب بالربع الأول والرابع } (\cos \frac{x}{2}) \quad (d)$$

$$40- \cos(x - \frac{\pi}{4}) - \sin(x - \frac{\pi}{4}) = 0, [0, 2\pi]$$

$$\cos(x - \frac{\pi}{4}) = \sin(x - \frac{\pi}{4}), (x = \frac{\pi}{2}, \frac{3\pi}{2}) \text{ مجموعة الحل هي نقاط تقاطع المنحنيين معا وهي}$$

(b)



السؤال الثاني:

$$1- \ln|\cos x| = \frac{1}{2}(\ln|1 + \cos 2x| - \ln|2|)$$

$$\Rightarrow \frac{1}{2}(\ln|1 + \cos 2x| - \ln|2|) = \frac{1}{2}(\ln \left| \frac{1 + \cos 2x}{2} \right|) = \frac{1}{2} \ln|\cos^2 x| = \frac{1}{2} \times 2 \ln|\cos x| = \ln|\cos x|$$

$$2- 4(\sin^6 x + \cos^6 x) = 4 - 3 \sin^2 2x$$

$$4(\sin^6 x + \cos^6 x) = 4(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x), \sin^2 x + \cos^2 x = 1$$

$$= 4(1)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) = 4((\sin^2 x)^2 - \sin^2 x \cos^2 x + (\cos^2 x)^2)$$

$$= 4\left(\left(\frac{1 - \cos 2x}{2}\right)^2 - \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right) + \left(\frac{1 + \cos 2x}{2}\right)^2\right), \sin^2 x = \frac{1 - \cos 2x}{2}, \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$= 4\left(\left(\frac{1 - 2\cos 2x + \cos^2 2x}{4}\right) - \left(\frac{1 - \cos^2 2x}{4}\right) + \left(\frac{1 + 2\cos 2x + \cos^2 2x}{4}\right)\right)$$

$$= 4\left(\frac{1 - 2\cos 2x + \cos^2 2x - 1 + \cos^2 2x + 1 + 2\cos 2x + \cos^2 2x}{4}\right)$$

$$= 4\left(\frac{\cos^2 2x + \cos^2 2x + 1 + \cos^2 2x}{4}\right) = 4\left(\frac{3\cos^2 2x + 1}{4}\right) = 4\left(\frac{1}{4} + \frac{3\cos^2 2x}{4}\right)$$

$$= 4\left(\frac{1}{4} + \frac{3}{4}(\cos^2 2x)\right) = 4\left(\frac{1}{4} + \frac{3}{4}(1 - \sin^2 2x)\right), \cos^2 2x = 1 - \sin^2 2x$$

$$= 4\left(\frac{1}{4} + \frac{3}{4} - \frac{3}{4}\sin^2 2x\right) = 4\left(1 - \frac{3}{4}\sin^2 2x\right) = 4 - \frac{12}{4}\sin^2 2x = 4 - 3\sin^2 2x$$

$$3- \tan^2 x \cos x = \tan x \sin x$$

$$\tan^2 x \cos x = \frac{\sin^2 x}{\cos^2 x} \cos x = \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \sin x = \tan x \sin x$$

$$4- \cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \frac{1 - \frac{\sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2}}}{1 + \frac{\sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2}}} = \frac{\frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2}}}{\frac{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2}}} = \left(\frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2}}\right) \times \left(\frac{\cos^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}}\right) = \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}} = \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{1}, \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = 1$$

$$= \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \cos 2 \frac{\theta}{2} = \cos \theta, \cos^2 \theta - \sin^2 \theta = \cos 2\theta$$

السؤال الثالث:

$$1- \sin x + 2 \sin x \cos x = 0, [0, 2\pi]$$

$$\sin x + 2 \sin x \cos x = 0 \Rightarrow \sin x(1 + 2 \cos x) = 0$$

$$\therefore \sin x = 0 \Rightarrow x = 0, \pi, 2\pi \in [0, 2\pi]$$

$$\text{Or } 1 + 2 \cos x = 0 \Rightarrow 2 \cos x = -1 \Rightarrow \cos x = -\frac{1}{2}$$

$$\Rightarrow \therefore \cos x = \left| -\frac{1}{2} \right| = \frac{1}{2} \Rightarrow x = 60^\circ = \frac{\pi}{3}, \text{ سالب بالربع الثاني والثالث } (\cos x)$$

$$\Rightarrow x = \pi - \frac{\pi}{3} = \frac{3\pi - \pi}{3} = \frac{2\pi}{3}, \text{ الربع الثاني}$$

$$\Rightarrow x = \pi + \frac{\pi}{3} = \frac{3\pi + \pi}{3} = \frac{4\pi}{3}, \text{ الربع الثالث } \Rightarrow \therefore x = \frac{2\pi}{3}, x = \frac{4\pi}{3} \in [0, 2\pi]$$

$$\therefore x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi \in [0, 2\pi]$$

$$2- \tan \frac{x}{2} - \sin x = 0, [0, 2\pi]$$

$$\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} - \sin x = 0 \Rightarrow \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} - 2 \sin \frac{x}{2} \cos \frac{x}{2} = 0, \sin 2x = 2 \sin x \cos x \Rightarrow \sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$\Rightarrow \frac{\sin \frac{x}{2} - 2 \sin \frac{x}{2} \cos^2 \frac{x}{2}}{\cos \frac{x}{2}} = 0 \Rightarrow \frac{\sin \frac{x}{2} (1 - 2 \cos^2 \frac{x}{2})}{\cos \frac{x}{2}} = 0 \Rightarrow \sin \frac{x}{2} (1 - 2 \cos^2 \frac{x}{2}) = 0$$

$$\therefore \sin \frac{x}{2} = 0 \Rightarrow \therefore \frac{x}{2} = 0, \pi, 2\pi \Rightarrow x = 0 \times 2 = 0 \in [0, 2\pi], x = \pi \times 2 = 2\pi \in [0, 2\pi],$$

$$x = 2\pi \times 2 = 4\pi \notin [0, 2\pi] \Rightarrow \therefore x = 0, 2\pi \in [0, 2\pi]$$

$$\text{or } 1 - 2 \cos^2 \frac{x}{2} = 0 \Rightarrow 1 = 2 \cos^2 \frac{x}{2} \Rightarrow \cos^2 \frac{x}{2} = \frac{1}{2} \Rightarrow \cos \frac{x}{2} = \pm \frac{1}{\sqrt{2}}, \text{ الزاوية } (\frac{x}{2}) \text{ تقع في جميع الارباع ,}$$

$$\Rightarrow \therefore \cos \frac{x}{2} = \left| -\frac{1}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}, \text{ موجب بالربع الاول والرابع , } (\cos \frac{x}{2})$$

$$\Rightarrow \frac{x}{2} = 45^\circ = \frac{\pi}{4} \Rightarrow x = 2 \times \frac{\pi}{4} = \frac{\pi}{2} \in [0, 2\pi], \frac{x}{2} = 2\pi - \frac{\pi}{4} = \frac{8\pi - \pi}{4} = \frac{7\pi}{4} \Rightarrow x = 2 \times \frac{7\pi}{4} = \frac{7\pi}{2} \notin [0, 2\pi]$$

$$\therefore x = \frac{\pi}{2} \in [0, 2\pi]$$

$$\Rightarrow \therefore \cos \frac{x}{2} = -\frac{1}{\sqrt{2}}, \text{ سالب بالربع الثاني والثالث , } (\cos \frac{x}{2})$$

$$\Rightarrow \frac{x}{2} = 135^\circ = \pi - \frac{\pi}{4} = \frac{4\pi - \pi}{4} = \frac{3\pi}{4} \Rightarrow x = 2 \times \frac{3\pi}{4} = \frac{3\pi}{2} \in [0, 2\pi], \frac{x}{2} = \pi + \frac{\pi}{4} = \frac{4\pi + \pi}{4} = \frac{5\pi}{4}$$

$$\Rightarrow x = 2 \times \frac{5\pi}{4} = \frac{5\pi}{2} \notin [0, 2\pi]$$

$$\therefore x = \frac{3\pi}{2} \in [0, 2\pi] \Rightarrow \therefore x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi \in [0, 2\pi]$$

$$3- \sin(\cos x) = 0, [0, 2\pi]$$

$$\sin(y) = 0, y = \cos x \Rightarrow \therefore y = 0, \pi, 2\pi \Rightarrow \therefore \cos x = 0 \Rightarrow \therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\therefore \cos x = \pi \Rightarrow \therefore \cos x = \phi, \text{ مدى } (\cos x) \text{ في الفترة } [-1, 1],$$

$$\therefore \cos x = 2\pi \Rightarrow \therefore \cos x = \phi, \text{ مدى } (\cos x) \text{ في الفترة } [-1, 1],$$

$$4- 4 \sin^2 x - 8 \sin x + 3 = 0, [0, 2\pi]$$

$$(2 \sin x - 1)(2 \sin x - 3) = 0$$

$$2 \sin x - 1 = 0 \Rightarrow 2 \sin x = 1 \Rightarrow \sin x = \frac{1}{2}, \text{ موجب بالربع الاول والثاني , } (\sin x)$$

$$\therefore x = \frac{\pi}{6}, \therefore x = \pi - \frac{\pi}{6} = \frac{6\pi - \pi}{6} = \frac{5\pi}{6} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \in [0, 2\pi]$$

$$\text{or } 2 \sin x - 3 = 0 \Rightarrow 2 \sin x = 3 \Rightarrow \sin x = \frac{3}{2} \Rightarrow \therefore \sin x = \phi, \text{ مدى } (\sin x) \text{ في الفترة } [-1, 1],$$

$$5- \cos^2 x + 3 \cos x + 2 = 0, [0, 2\pi]$$

$$(\cos x + 2)(\cos x + 1) = 0$$

$$\cos x + 2 = 0 \Rightarrow \cos x = -2 \Rightarrow \therefore \cos x = \phi, \text{ مدى } (\cos x) \text{ في الفترة } [-1, 1],$$

$$\text{or } \cos x + 1 = 0 \Rightarrow \cos x = -1 \Rightarrow \therefore x = -\pi \in [0, 2\pi]$$

$$6- 2 \cos^2 x + 3 \sin x = 0, [0, 2\pi]$$

$$2(1 - \sin^2 x) + 3 \sin x = 0, \cos^2 x = 1 - \sin^2 x$$

$$2 - 2 \sin^2 x + 3 \sin x = 0 \Rightarrow -2 + 2 \sin^2 x - 3 \sin x = 0 \text{ (-1) بالضرب في}$$

$$2 \sin^2 x - 3 \sin x - 2 = 0 \Rightarrow (2 \sin x + 1)(\sin x - 2) = 0$$

$$2 \sin x + 1 = 0 \Rightarrow 2 \sin x = -1 \Rightarrow \sin x = -\frac{1}{2} \Rightarrow \sin x = \left| -\frac{1}{2} \right| = \frac{1}{2} \therefore x = \frac{\pi}{6},$$

$$\text{سالب بالربع الثالث والرابع } (\sin x), \therefore x = \pi + \frac{\pi}{6} = \frac{6\pi + \pi}{6} = \frac{7\pi}{6}, \therefore x = 2\pi - \frac{\pi}{6} = \frac{12\pi - \pi}{6} = \frac{11\pi}{6}$$

$$\Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6} \in [0, 2\pi]$$

$$\sin x - 2 = 0 \Rightarrow \sin x = 2 \Rightarrow \therefore \sin x = \phi, [-1, 1] \text{ في الفترة } (\sin x) \text{ مدى}$$